PG – III Sem

Math – 531

2018

Full Marks : 80

Time : 3 hours

The figures in the right-hand margin indicate marks

Answer **all** questions

1. *(a)* Define the terms : 6

(i) Feasible solution

(ii) Basic feasible solution

(iii) Degenerate basic feasible solution

(iv) Feasible region.

(b) Explain the graphical solution of an LPP. What is its limitation? 5

(c) Using two-phase method, solve the following LPP : 9

$$\text{Max } Z = 5x_1 - 4x_2 + 3x_3$$

subject to the constraints

$$2x_1 + x_2 - 6x_3 = 20$$

$$6x_1 + 5x_2 + 10x_3 \le 76$$

$$8x_1 - 3x_2 + 6x_3 \le 50$$

$$x_1, x_2, x_3 \ge 0$$

Or

(d) Prove that the set of feasible solutions to an LPP is a convex set. 4

(e) Differentiate between penalty method and two-phase method. 5

(f) Explain the different canonical forms of an LPP. 4

(g) Solve the following LPP : 7

$$\text{Min } Z = 2x_1 + x_2$$

subject to the constraints

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

2. (a) Define primal and dual problem. Find the dual problem of a general transportation problem. State and prove the fundamental theorem of duality. 10

(b) Find the optimum solution of the following transportation problem : 10

90	90	100	100	200
50	70	130	85	100
75	100	100	30	

Or

(c) Differentiate between a transportation problem and an assignment problem. 5

(d) Explain a method to find an optimum solution of an assignment problem. 7

(e) Solve the following assignment problem : 8

	A	*B*	*C*	*D*	*E*
1	16	13	17	19	20
2	14	12	13	16	17
3	14	11	12	17	18
4	5	5	8	8	11
5	5	3	8	8	10

3. (a) Discuss a method of post-optimal problem when there is variation in the cost vector. 8

(b) Consider the LPP

$$\text{Max } Z = -x_1 + 2x_2 - x_3$$

subject to the constraints

$$3x_1 + x_2 - x_3 \leq 10$$
$$-x_1 + 4x_2 + x_3 \geq 6$$
$$x_1 + x_2 \leq 4$$
$$x_1, x_2, x_3 \geq 0$$

Determine the ranges for discrete changes in the components b_2, b_3 of the requirement vector so as to maintain the feasibility of the current optimum solution. 12

Or

(c) Explain the parametric linear programming problem when the cost vector is parameter. What happens if the requirement vector is parameter? 10

(d) Consider the parametric linear programming problem

$$\text{Max } Z = (6-\lambda)\, x_1 + (12-\lambda)\, x_2 + (4-\lambda)\, x_3$$

subject to the constraints

$$3x_1 + 4x_2 + x_3 \le 2$$
$$x_1 + 3x_2 + 2x_3 \le 1$$
$$x_1, x_2, x_3 \ge 0$$

Perform complete parametric programming analysis and identify all critical values of λ over which the solution remains basic feasible and optimal. 10

4. *(a)* Explain the following terms : 5

(i) Game

(ii) Strategy

(iii) Two-person zero-sum game.

(b) Let (a_{ij}) be the $m \times n$ payoff matrix for a two-person zero-sum game. If $\underline{v}$ denotes the maximin value and $\overline{v}$ denotes the minimax value of the game, then show that $\overline{v} \ge \underline{v}$. 5

(c) Two companies A and B are competing for the same product. Their different strategies are given in the following payoff matrix :

$$\begin{array}{cc} & \textit{Company } A \\ \textit{Company } B & \begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \end{bmatrix} \end{array}$$

Use LPP method and determine the best strategies for both the companies. 10

Or

(d) Explain the maximin-minimax principle for a game of two players. Can we apply for $(m \times n)$ game? Justify your answer. 4

(e) State and explain a method to find the value of a 2×2 two-person zero-sum game without any saddle point. 8

(f) Solve the game graphically whose payoff matrix is given by

$$\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$$

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1. *(a)* Define the sequence space $l_p, 1 \le p < \infty$. Show that the space $(l_p, d_p), 1 \le p < \infty$ is separable. 10

(b) Define the function space L_p on a measurable set E. Let $E = \mathbb{R}$ and

$$f(x) = \begin{cases} \dfrac{1}{\sqrt{|x|}}, & \text{if } 0 < |x| < 1 \\ 0, & \text{if else} \end{cases}$$

$$g(x) = \begin{cases} 0, & \text{if } |x| < 1 \\ \dfrac{1}{x}, & \text{if else} \end{cases}$$

Show that $f \in L^1(\mathbb{R})$, $f \notin L^2(\mathbb{R})$, $g \in L^2(\mathbb{R})$ and $g \notin L^1(\mathbb{R})$. 10

Or

(c) Define continuous linear operator. Show that on a finite-dimensional normed space, every linear operator is continuous. Give an example of an non-continuous linear operator. 10

(d) Prove that every inner product space is a norm linear space and every norm linear space is a metric space. Give an example of a metric space, which is not a norm linear space. 10

2. (a) If Y be any closed subspace of a Hilbert space H, then prove that $H = Y \oplus Y^{\perp}$, where $Y^{\perp}$ is the orthogonal complement of Y. 10

(b) Describe Gram-Schmidt ortho-normalization process. Let $x_1(t) = t^2$, $x_2(t) = t$ and $x_3(t) = 1$. Orthonormalize x_1, x_2, x_3 on the interval $[-1, 1]$, where

$$\langle x, y \rangle = \int_{-1}^{1} x(t)\, y(t)\, dt$$ 10

Or

(c) Prove that the Hilbert adjoint operator T^* of a bounded linear operator T from a Hilbert space to another Hilbert space exists, is unique and is a bounded linear operator with norm $\|T\| = \|T^*\|$. 10

(d) State Bessel's inequality. Let H be a Hilbert space and E is an orthonormal basis of X. Then show that every $x \in X$ can be represented as

$$x = \sum_{u \in E} \langle x, u \rangle\, u$$ 10

3. (a) Define algebraic reflexive space. Show that the dual of $\mathbb{R}^n$ is $\mathbb{R}^n$. 10

(b) Define second category space. Prove that a complete metric space is of second category. 10

Or

(c) State and prove Hahn-Banach extension theorem on real vector space. 10

(d) Define graph of a linear operator T. Let $T : D(T) \to Y$, domain of $T = D(T) \subset X$, where X and Y are normed spaces. Show that T is a closed operator, if and only if graph of T is a closed subspace of $X \times Y$. 10

4. *(a)* Define contraction mapping. State and prove Banach-Contraction principle. 10

(b) Define compact operator. Let $X = L^2[a, b]$ and $Y = C[a, b]$. Consider the integral operator

$$Tx(s) = \int_a^b k(s, t)\, x(t)\, dt$$

where $k(s, t) \in C[a, b] \times [a, b]$. Then show that T is a compact operator on X. 10

Or

(c) Define resolvent set and spectrum of a linear operator. Prove that the spectrum of a bounded linear operator on a Banach space is closed. 10

(d) *(i)* Let $A = B(X, Y)$ and $B = B(Y, Z)$. If one of them is compact, then show that BA is a compact operator from X to Z. 5

(ii) Prove that every linear operator on a finite-dimensional normed space is compact. 5

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The questions are of equal value

Answer **one** question from each Unit

Symbols carry their usual meanings

Unit—I

1. *(a)* Find a general integral of *any two* of the following p.d.e.s :

(i) $p\cos(x+y)+q\sin(x+y)=z$

(ii) $(3x+y-z)\,p+(x+y-z)\,q = 2(z-y)$

(iii) $(y^2+z^2-x^2)\,p-2xyq+2zx=0$

(b) Find the complete integral of the equation $p^2x+q^2y-z=0$ by Jacobi method.

2. *(a)* Find the complete integral of *any two* of the following by Charpit method :

(i) $z^2-pqxy=0$

(ii) $xpq - yq^2 - 1 = 0$

(iii) $2z + p^2 = qy + 2y^2 = 0$

(b) Define compatibility of system of two partial differential equations of first order. Show that the equations $xp - yq - x = 0$ and $x^2 p + q - xz = 0$ are compatible and hence find a one parameter family of common solutions.

Unit—II

3. (a) What is Cauchy problem for a first-order quasilinear p.d.e.? Explain your answer. Solve the partial differential equation $uu_x + u_y = 0$ with Cauchy data $u(x, 0) = x, 0 \le x \le 1$.

(b) Find the equation of the integral surface of the equation

$$x^3 p + y(3x^2 + y) q = z(2x^2 + y)$$

which passes through the curve $x_0 = 1$, $y_0 = s$, $z_0 = s(1+s)$.

4. (a) Find the complete integral of the equation $(p^2 + q^2) x = pz$ and the integral surface corresponding to the initial data curve Γ given by $x_0 = 0$, $y_0 = s^2$, $z_0 = 2s$.

(b) Find the general integral of the equation

$$(2x - y)y^2 u_x + 8(y - 2x) x^2 u_y = 2(4x^2 + y^2) y$$

and deduce the solution of the Cauchy problem when $u(x, 0) = \frac{1}{2x}$ on a portion of the x-axis.

Unit—III

5. (a) Classify and reduce the partial differential equation

$$y^2 u_{xx} - 2xyu_{xy} + x^2 u_{yy} = \frac{y^2}{x} u_x + \frac{x^2}{y} u_y$$

to a canonical form and solve it.

(b) Find the solution of the wave equation

$$u_{tt} = c^2 u_{xx} \qquad u(x,t)$$

with the conditions $u(0, t) = 0$, $0 \le x \le L, t > 0$ by variable separable method.

6. (a) Solve the partial differential equation

$$u_{xx} = c^2 u_{tt} \quad 0 < x < \infty, \ t \ge 0$$

with initial condition $u(x, 0) = \eta(x)$, $u_t(x, 0) = v(x)$ and boundary condition $u(0, t) = 0$, $t \ge 0$.

(b) Find the solution of the wave equation $u_{tt} = c^2 u_{xx}$ with the conditions—

(i) $u(0, t) = u(2, t) = 0$

(ii) $u(x, 0) = \sin^3 \dfrac{\pi x}{2}$

(iii) $u_t(x, 0) = 0$

Unit—IV

7. (a) Solve the Laplace equation $u_{xx} + u_{yy} = 0, \; -\infty < x < \infty, \; y \geq 0,$ with $u(x, 0) = f(x), \; -\infty < x < \infty$ and u is bounded as $y \to \infty$, when u, u_x vanish as $|x| \to \infty$.

(b) Describe the various boundary value problems for a Laplace equation. Prove that if $u(x, y)$ is harmonic in a bounded domain Ω and is continuous in $\bar{\Omega}$, then u attains its maximum and minimum on the boundary $\partial\Omega$.

8 (a) Find the solution of interior Dirichlet boundary value problem for a circle of radius r.

(b) Obtain the solution of the initial value problem of heat conduction in an infinite rod given by

$$u_t = k u_{xx}, \qquad -\infty < x < \infty, \; t > 0$$
$$u(x, 0) = f(x), \quad -\infty < x < \infty$$

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Answer **all** questions

1. *(a)* Find the half-range series for $f(x) = x^3$, $0 < x < L$. 10

(b) State and prove the Riemann-Lebesgue lemma. 10

Or

(c) Expand the function $f(x) = x^2$, $-\pi < x < \pi$ as a series and hence prove that

$$\frac{\pi^2}{12} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots$$ 10

(d) State and prove Dirichlet's pointwise convergence theorem. 10

2. *(a)* Find the Fourier series of $f(x) = \pi x$, $x \in [-2, 2]$ and hence prove that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$ 10

(b) Prove that the series

$$1-2+3-4+\cdots=\sum_{n\in N}(-1)^{n+1}\cdot n$$

is not $(c, 1)$ summable. 10

Or

(c) Find the Fourier series of $f(x)=x|x|$, $-2<x<2$. 8

(d) If the Fourier series

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$$

converges on a set E of positive Lebesgue measure, then prove that $a_n, b_n\to 0$. 12

3. (a) Let $f(x)$ be continuous and integrable on x-axis and $f'(x)$ be piecewise continuous on every finite interval and let $f(x)\to 0$ as $x\to\infty$. Then prove that—

(i) $F_c[f'(x)]=wF_s[f(x)]-\sqrt{\frac{2}{\pi}}f(0)$

(ii) $F_s[f'(x)]=-wF_c[f(x)]$

where F_s and F_c are Fourier sine and cosine transform. 8

(b) Prove the following convolutions for the functions : 12

(i) $f*g=g*f$

(ii) $f*(g*h)=(f*g)*h$

(iii) $f*(g+h)=f*g+f*h$

Or

(c) If $f(x)$ is continuous and integrable on x-axis and $f'(x)$ is piecewise continuous on every finite interval and let $f(x)\to 0$ as $x\to\infty$. Then prove that—

(i) $F_c[f''(x)]$

$$=-w^2F_c[f(x)]-\sqrt{\frac{2}{\pi}}f'(0)$$

(ii) $F_s[f''(x)]$

$$=-w^2F_s[f(x)]+\sqrt{\frac{2}{\pi}}wf(0)$$

12

(d) Find the cosine transform of

$$f(x)=\begin{cases}1 & \text{if } 0<x<1\\ -1 & \text{if } 1<x<2\\ 0 & \text{if } x>2\end{cases}$$

and also obtain the inverse cosine transform. 8

4. *(a)* State and prove Parseval's identities. 10

(b) Find the Fourier transform of

$$f(x) = \begin{cases} |x| & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Also find the inverse Fourier transform. 10

Or

(c) Define Discrete Fourier Transform and prove inversion theorem for the DFT. 10

(d) Prove that

$$\int_0^\infty \frac{\sin^4 w}{w^4}\, dw = \frac{\pi}{3}$$ 10

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Answer **one** question from each Unit

Unit—I

1. Define Poisson integral. Prove that the Poisson kernel can be represented by

$$P_r(\theta - t) = \frac{1 - r^2}{1 - 2r\cos(\theta - t)\, t r^2}$$

2. Suppose u is a continuous real function on the closed unit disc $\overline{U}$ and suppose u is harmonic in U. Then prove that u is the Poisson integral of its restriction to T and u is the real part of the holomorphic function

$$f(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{e^{it} + z}{e^{it} - z}\, u(e^{it})\, dt, \quad (z \in U)$$

3. State and prove Harnack's theorem for harmonic function.

Unit—II

4. If f is an entire function of finite order λ, then prove that f has finite genus $\mu \leq \lambda$.

5. Define zeros of entire function with illustrations. Define order of entire function and prove Poincare theorem.

6. If n is a positive integer and

$$P(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0$$

where $a_0, a_1, \cdots, a_{n-1}$ are complex numbers, then prove that P has precisely n zeros in the plane.

Unit—III

7. Prove that

$$\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)=\frac{1}{2}$$

8. Define meromorphic function with suitable example. Prove that

$$\lim_{A \to \infty} \int_{-A}^{A} \frac{\sin x}{x} \cdot e^{itx} \cdot dx = \begin{cases} \pi & \text{if } -1 < t < 1 \\ 0 & \text{if } |t| > 1 \end{cases}$$

9. What do you mean by analytic continuity of Zeta function? Prove Jensen's formula.

Unit—IV

10. Define elliptic function with example. Prove that every elliptic function without having singularities is a constant function.

11. State and prove addition theorem for P function.

12. Define elliptic modular function with example and state all general properties of elliptic functions.

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